

A note on product-convolution for generalized subexponential distributions

Dimitrios Konstantinides^a, Remigijus Leipus^b, Jonas Šiaulys^c,

^aDepartment of Statistics and Actuarial - Financial Mathematics, University of the Aegean, Karlovassi, GR-83 200 Samos, Greece

^bInstitute of Applied Mathematics, Vilnius University, Naugarduko 24, Vilnius LT-03225, Lithuania

^cInstitute of Mathematics, Vilnius University, Naugarduko 24, Vilnius LT-03225, Lithuania

Abstract

In this paper we consider the stability property of the class of generalized subexponential distributions with respect to product-convolution. Assuming that the primary distribution is in the class of generalized subexponential distributions, we find conditions for the second distribution in order that that their product-convolution belongs to the class of generalized subexponential distributions as well. The similar problem for the class of generalized subexponential positively decreasing-tailed distributions is considered.

Keywords: product-convolution, subexponential distribution, generalized subexponential distribution, positively decreasing-tailed distribution

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References

- [1] Albin, J.M.P., Sundén, M., 2009. On the asymptotic behavior of Lévy processes, Part I: Subexponential and exponential processes. *Stoch. Proc. Appl.* 119, 281–304.
- [2] Assmussen, S., 2003. *Applied Probability and Queues*. 2nd ed. Springer, New York.
- [3] Assmussen, S., Albrecher, H., 2010. *Ruin Probabilities*. World Scientific, Singapore.
- [4] Baltrūnas, A., Omey, E., Van Gulck, S., 2006. Hazard rates and subexponential distributions. *Publ. de l’Institute Math.* 80, 29–46.
- [5] Bardoutsos, A.G., Konstantinides, D.G., 2011. Characterization of tails through hazard rate and convolution closure properties. *J. Appl. Probab.* 48A, 123–132.
- [6] Berkes, I., Györfi, L., Kevei P., 2017. Tail probabilities of St. Petersburg sums, trimmed sums, and their limit. *J. Theor. Probab.* 30, 1104–1129.
- [7] Bingham, N.H., Goldie, C.M., Teugels, J.L., 1987. *Regular Variation*. Cambridge University Press, Cambridge.
- [8] Chen Y., Ng K.W., Tang Q., 2005. Weighted sums of subexponential random variables and their maxima. *Adv. Appl. Probab.* 37, 510–522.
- [9] Cheng, D., Wang, Y., 2012. Asymptotic behavior of the ratio of tail probabilities of sum and maximum of independent random variables. *Lith. Math. J.* 52, 29–39.
- [10] Chistyakov, V.P., 1964. A theorem on sums of independent, positive random variables and its applications to branching processes. *Theory Probab. Appl.* 9, 640–648.
- [11] Cline, D.B.H., 1986. Convolution tails, product tails and domain of attraction. *Probab. Theory Relat. Fields* 72, 529–557.
- [12] Cline, D.B.H., Samorodnitsky, G., 1994. Subexponentiality of the product of independent random variables. *Stoch. Proc. Appl.* 49, 75–98.
- [13] Cui, Z., Wang, Y., 2020. On the long tail property of product convolution. *Lith. Math. J.* 60, 315–329.
- [14] Dirma, M., Paukštys, S., Šiaulys, J., 2021. Tails of the moments for sums with dominatedly varying random summands, *Mathematics* 9, 824.
- [15] Embrechts, P., Klüppelberg, C., Mikosch, T., 1997. *Modelling Extremal Events: for Insurance and Finance*. Springer, New York.
- [16] Feller, W., 1969. One-sided analogues of Karamata’s regular variation. *L’Enseignement Mathématique* 15, 107–121.
- [17] Foss, S., Korshunov, D., Zachary, S., 2013. *An Introduction to Heavy-Tailed and Subexponential Distributions*. 2nd edition, Springer-Verlag, New York.
- [18] Galambos, J., Simonelli, I., 2004. *Product of Random Variables: Applications to Problems of Physics and to Arithmetical Functions*. CRC Press, Boca Raton, London, New York.
- [19] Gerber, H.U., 1979. *An Introduction to Mathematical Risk Theory*. Huebner Foundation Monographs, Philadelphia.
- [20] Goldie, C.M., 1978. Subexponential distributions and dominated variation tails. *J. Appl. Probab.* 15, 440–442.

Email addresses: konstant@aegean.gr (Dimitrios Konstantinides), remigijus.leipus@mif.vu.lt (Remigijus Leipus), jonas.siaulys@mif.vu.lt (Jonas Šiaulys)

- [21] Hao, X., Tang, Q., 2008. A uniform asymptotic estimate for discounted aggregate claims with subexponential tails. *Insur. Math. Econ.* 43, 116–120.
- [22] Klüppelberg, C. 1989. Subexponential distributions and characterization of related classes. *Probab. Theor. Rel. Fields* 82, 259–269.
- [23] Klüppelberg, C. 1990. Asymptotic ordering of distribution functions and convolution semigroups. *Semigroup Forum* 40, 77–92.
- [24] Konstantinides, D.G., 2008. A class of heavy tailed distributions. *J. Numer. Appl. Math.* 96, 127–138.
- [25] Konstantinides, D.G., Tang, Q., Tsitsiashvili, G., 2002. Estimates for the ruin probability in the classical risk model with constant interest force in the presence of heavy tails. *Insur. Math. Econ.* 31, 447–460.
- [26] Leipus, R., Paukštys, S., Šiaulys, J.. 2021. Tails of higher-order moments of sums with heavy-tailed increments and application to the Haezendonck-Goovaerts risk measure. *Stat. Probab. Lett.* 170, 108998.
- [27] Lin, J., Wang, Y., 2012. New examples of heavy-tailed O-subexponential distributions and related closure properties. *Stat. Probab. Lett.* 82, 427–432.
- [28] Markovich N., 2007. Nonparametric Analysis of Univariate Heavy-Tailed Data. Wiley. New York.
- [29] Matuszewska, W., 1964. On generalization of regularly increasing functions. *Studia Mathematica* 24, 271–279.
- [30] Rolski, T., Schmidli, H., Schmidt, V., Teugels, J.F., 1999. Stochastic Processes for Insurance and Finance. Wiley. Chichester.
- [31] Schmidli, H., 2017. Risk Theory. Springer, New York.
- [32] Shimura, T., Watanabe, T., 2005. Infinite divisibility and generalized subexponentiality. *Bernoulli* 11, 445–469.
- [33] Tang, Q., 2004. Asymptotics for the finite time ruin probability in the renewal model with consistent variation. *Stoch. Models* 20, 281–297.
- [34] Tang, Q., 2006. On convolution equivalence with applications. *Bernoulli* 12, 535–549.
- [35] Tang, Q., 2006. The subexponentiality of products revisited. *Extremes* 9, 231–241.
- [36] Tang Q., Tsitsiashvili G., 2003. Precise estimates for the ruin probability in finite horizon in a discrete-time model with heavy-tailed insurance and financial risks, *Stoch. Process. Appl.* 108, 299–325.
- [37] Tang, Q., Tsitsiashvili, G., 2003. Randomly weighted sums of subexponential random variables with application to ruin theory, *Extremes* 6, 171–188.
- [38] Tang, Q., Tsitsiashvili, G., 2004. Finite- and infinite-time ruin probabilities in the presence of stochastic returns on investments, *Adv. Appl. Probab.* 36, 1278–1299.
- [39] Watanabe, T., 2008. Convolution equivalence and distributions of random sums, *Probab. Theory Relat. Fields*. 142, 367–397.
- [40] Watanabe, T., Yamamoto, K., 2010. Ratio of the tail of an infinitely divisible distribution on the line to that of its Lévy measure. *Electron. J. Probab.* 15, 44–74.
- [41] Xu, H., Foss, S., Wang, Y., 2015. Convolution and convolution-root properties of long-tailed distributions, *Extremes* 18, 605–628.
- [42] Zwart, A.P., 2001. Queueing systems with heavy tails. Technische Universiteit Eindhoven. <https://doi.org/10.6100/IR547196>