Optimal Reinsurance Strategies in a Partially Observable Catastrophic Framework

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Abstract

Optimal reinsurance-investment problems have been deserved special attention during the past few years and they have been investigated in many different model settings. Insurance companies can hardly deal with all the different sources of risk existing in the real world, so they try to hedge against at least part of them, by re-insuring with other institutions. Large part of the literature available focuses mainly on classical reinsurance contracts such as the proportional and the excess-of-loss, which were extensively investigated under a variety of optimization criteria, see Brachetta and Ceci [4], Irgens and Paulsen [9] and Liu and Ma [12]. Some of the classical papers devoted to the subject assume a diffusion type dynamics for the surplus process, while the more recent literature considers surplus processes including jumps.

Recently Cao, Landriault and Li [6] investigated the optimal reinsurance-investment problem in the model setting proposed by Dassios and Zhao [8] with a reward function of mean-variance type. This particular choice of the reward function raises the problem of time-inconsistency and the failing of the dynamic programming principle, so the authors attack the optimization problem by describing the decision-making process as a non-cooperative game against all strategies adopted by future players.

A different line of research related to the optimal-reinsurance investment problem focuses on the possibility that the insurer does not have access to all the information necessary to choose the reinsurance strategy and it deals with the stochastic optimization problem under partial information. Liang and Bayraktar [11] were the first to introduce a partial information framework in optimal reinsurance problems. They consider the optimal reinsurance and investment problem in an unobservable Markov-modulated compound Poisson risk model, where the intensity and jump size distribution are not known but have to be inferred from the observations of claim arrivals. Ceci, Colaneri and Cretarola [7] derive risk-minimizing investment strategies

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when information available to investors is restricted and they provide optimal hedging strategies for unit-linked life insurance contracts. Jang, Kim and Lee [10] present a systematic comparison between optimal reinsurance strategies in complete and partial information framework and quantify the information value in a diffusion setting.

More recently Brachetta and Ceci [5] investigate the optimal reinsurance problem under the criterion of maximizing the expected (exponential)utility of terminal wealth when the insurance company has restricted information on the loss process in a model with claim arrival intensity and claim sizes distribution affected by an unobservable environmental stochastic factor. By filtering techniques (with marked point process observations), they reduce the original problem to an equivalent stochastic control problem under full information. Since the classical Hamilton-Jacobi-Bellman approach does not apply in this setting, due to the infinite dimensionality of the filter, they choose an alternative approach based on Backward Stochastic Differential Equations (henceforth BSDEs) and they characterize the value process and the optimal reinsurance strategy in terms of the unique solution to a BSDE driven by a marked point process.

In the present paper we investigate the optimal reinsurance strategy for a risk model with jump clustering properties in a partial information setting. The model we consider is very similar to the model proposed Dassios and Zhao [8], but differently from Cao et Al. [6] our utility function is of exponential type and the investor has only partial information available, more precisely the insurer can only observe the cumulative claim process. The externally-excited component of the intensity is not observable and the insurer needs to estimate the stochastic intensity by solving a filtering problem. In a partially observable framework, our goal is to characterize the value process and the optimal strategy.

Keywords: Optimal reinsurance; Partial information; Hawkes processes; Cox processes with shot noise; BSDEs; Risk measures.

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