Bowley vs. Pareto Optima in Reinsurance Contracting

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Abstract

The notion of a Bowley optimum has gained recent popularity as an equilibrium concept in problems of risk sharing and optimal reinsurance. In this paper, we examine the relationship between Bowley optimality and Pareto efficiency in a problem of optimal reinsurance, under fairly general preferences. Specifically, while we show that Bowley-optimal contracts are indeed Pareto efficient (hence providing a first welfare theorem), we also show that only those Pareto-efficient contracts that make the insurer indifferent between suffering the loss and entering into the reinsurance contract are Bowley optimal (hence providing only a partial second welfare theorem). We interpret the latter result as indicative of the limitations of Bowley optimality as an equilibrium concept in this literature. We also discuss relationships with competitive equilibria in the context of convex distortion risk measures, and we provide an illustrative example for the case of Tail Value-at-Risk.

In the context of optimal contract design in reinsurance markets with symmetric information, Bowley solutions follow from a sequential procedure: (i) First, the reinsurer selects a pricing kernel, and in response, the insurer will select the indemnity function that minimizes their risk exposure given that pricing kernel; and (ii) second, knowing the insurer's demand as a function of the pricing kernel, the reinsurer then selects the pricing kernel that minimizes their risk exposure. Bowley solutions were first introduced by Bowley [5] in the context of a bilateral monopoly, and then first applied to optimal reinsurance design by Chan and Gerber [6]. This paper aims to examine micro-equilibrium properties of Bowley solutions in problems of optimal reinsurance contracting. Specifically, we study the relationship between Bowley optimality and Pareto efficiency for a broad class of risk measures, since the latter is the standard notion of optimality typically used in the related literature.

Chan and Gerber [6] characterize Bowley solutions when the insurer and reinsurer are risk-averse Expected-Utility (EU) maximizers. These results are extended to more general risk exchanges

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by Taylor [12]. Bowley solutions were then largely ignored in the literature until the recent work of Cheung et al. [8], who focused on preferences given by distortion risk measures rather than EU-preferences. The work of Cheung et al. [8] has reignited the interest in the Bowley solution as an optimality concept in the optimal reinsurance literature. Indeed, Li and Young [11] study Bowley solutions with preferences and premiums given by a mean-variance form, and Boonen et al. [2, 4] study Bowley solutions with asymmetric information about the preferences of the insurer. Chi et al. [9] construct a sequential game inspired by Bowley solutions, in which the reinsurer determines the premium budget and the insurer optimizes an EU objective under constraints on the first two moments of the indemnity.

Related to Bowley solutions, Stackelberg equilibria have gained popularity in industrial economics. In a Stackelberg equilibrium, two competitive firms compete in setting quantities in a duopoly. As a key difference with Bowley solutions, the two firms both set their quantities, and the policyholders are jointly modelled via an inverse demand function that leads to the price. On the other hand, in a Bowley solution, the monopolistic leader (reinsurer) and follower (insurer) bargain with each other about a reinsurance contract, which consists of an indemnity and a corresponding premium. Subsequently, the monopolistic leader sets the prices and the follower selects the optimal indemnity. Using this terminology, the approaches of Chen and Shen [7], Gavagan et al. [10], and Yuan et al. [13] are closer to a Bowley solution than to a Stackelberg equilibrium.

The present work is closest in spirit to Cheung et al. [8], but our main objective as well as our class of preferences are different. Our focus is on preferences that are translation-invariant, and sometimes also assumed convex, comonotonic-additive, and/or continuous; while the focus in Cheung et al. [8] is on the smaller class of convex distortion risk measures (i.e., concave distortion functions). Moreover, whereas the objective of Cheung et al. [8] is to construct Bowley solutions explicitly, our primary concern is deriving some key properties of Bowley solutions and examining how they relate to Pareto optima. Specifically, we show that Bowley solutions lead to Pareto-efficient contracts, but only those Pareto-optimal solutions that make the insurer indifferent with the status quo are Bowley solutions . In other words, Bowley solutions are precisely the solutions that are Pareto optimal and make the insurer indifferent with the status quo. While Pareto optimality is a reasonable and well-accepted efficiency property, the indifference of the insurer with the status quo can be perceived as undesirable. This paper thus also aims to provide a warning about the applicability of Bowley solutions. In fact, if one wishes to design a market mechanism such that the insurer strictly benefits from purchasing reinsurer, Bowley solutions may not suffice. Instead, complete market and comonotone market competitive equilibria (as in Boonen et al. [1, 3]) lead to Pareto optima in which the insurer may strictly benefit from the reinsurance arrangement. Additionally, one could consider the symmetric or asymmetric Nash bargaining solution, which we do not address here.

Keywords: Optimal Reinsurance, Pareto Optimality, Bowley Optimality, Convex Risk Measures, Distortion Risk Measures, TVaR.

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