On the Role of Skewness and Kurtosis in Tempered Stable and Other Lévy Models in Finance

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Abstract

We study the structure and properties of an infinite activity CGMY Lévy process X with given skewness S and kurtosis K of $X(1)$, without a Brownian component but allowing a drift component. The jump part of such a process is specified by the Lévy density which is Ce^{-Mx}/x^{1+Y} for $x > 0$ and $Ce^{-G|x|}/|x|^{1+Y}$ for $x < 0$. A main finding is that the quantity $R = S^2/K$ plays a major role, and that the class of CGMY processes can be parametrized by the mean EX(1), the variance Var$X(1)$, S, K and Y where Y varies in $[0, Y_{\max})$ with $Y_{\max} = (2-3R)/(1-R)$. Limit theorems for X are given in various settings, with particular attention to X approaching Brownian with drift, corresponding to the Black-Scholes model; for this sufficient conditions in a general Lévy process set-up are that $K \to 0$, or, in the spectrally positive case, that $S \to 0$. Implications for moment fitting of log-returns data are discussed. The paper also exploits the structure of spectrally positive CGMY processes as exponential tiltings (Esscher transforms) of stable processes, with the purpose of providing simple formulas for the log-returns density $f(x)$, short derivations of its asymptotic form, and quick algorithms for simulation and maximum likelihood estimation.

Keywords: Cumulant; Functional limit theorem; Log-return distribution; Exponentially tilted stable distribution; Moment method

References

[1] S. Asmussen (2022), "On the role of skewness and kurtosis in tempered stable (CGMY) Lévy models in finance " *Finance and Stochastics* (pending revision).

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