Robust Assessment of Life Insurance Products

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Abstract

The actuarial literature related to model risk has experienced a rapid growth in recent years. The idea behind this field of research is that any actuarial evaluation is prone to error in that the underlying loss distribution is typically only partially known. There is an extensive literature on finding bounds on a risk measure, see for example [1]. Clearly, a main quantity of actuarial interest that can be affected by model misspecification is the premium to be paid for an insurance contract. The net premium of a life-insurance contract depends essentially on two ingredients: the residual lifetime distribution function and the discount curve (financial risk).

The goal of the present paper is to develop a framework that can help the insurer to deal with model risk that arises from a misspecified residual lifetime distribution function. Recent studies have shown that in a low-interest rate environment longevity risk becomes the major risk-driver of the life-insurance business; see [5] and some of the references therein. In the life insurance literature, the study of the effect of model misspecification on the price of a contract is usually conducted using a parametric approach: it is assumed that the residual lifetime distribution belongs to one or more given families of probability distributions with uncertain parameters. [2] and [3] offer a discussion of this approach and provide some case studies. We propose to tackle this problem using a non-parametric approach.

Our analysis deals with the case in which the net premium π of a life insurance contract is computed according to the equivalence principle, i.e, $\pi = E(g(K_x))$, in which K_x describes the curtate residual lifetime of an x years old insured and g describes the discounted payoff function of the insurance contract. Mathematically, our goal is to study bounds for $E(g(K_x))$, when the estimated distribution of K_x is not fully trusted. The estimated probability distribution will be considered as reference distribution function. One can wonder about the range of $E(g(K_x))$ when we consider all the distribution functions that are somehow close to the reference distribution function, but not necessarily coming from the same parametric family. In order to formalize the notion of closeness between probability distributions we make use of the L^2 metric between two distribution functions (in a similar spirit as in [4], in which the Wasserstein distance is considered). In other words, we look for the best- and worst-case scenarios obtainable for π

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when considering all distributions having an L^2-distance from the reference distribution lower than a certain threshold.

Using this probability metric, we are able to reformulate the problem of computing the lower- and upper-bound for the net premium as a convex Quadratically Constrained Linear Program. This allow us to exploit the numerical tractability of convex linear programs to study the net premium bounds and their optimizing distributions from several points of view.

We further study the properties of this linear program to derive interesting properties of the net premium bounds and to show that in some cases explicit formulas for the optimizing distribution functions can be obtained. Our analysis highlight the convenience of using the L^2 distance to describe distributional uncertainty in the life insurance context. We provide several numerical examples that illustrate how our results can be used to obtain a robust assessment of an annuity contract. Specifically, the examples we provide show how our results can be useful in studying the net premium bounds, the probability distributions attaining the bounds, the relationship between model risk and interest rates, and some robust expected discounted utility maximization problems. Finally, we show that additional constraints, such as a the important case of a unimodality restriction, can be easily incorporated in our framework.

Keywords: Model Risk, Longevity Risk, Insurance pricing, L^p distance, Actuarial value.

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