Minimising the Weighted Time in Drawdown by Proportional Reinsurance

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Abstract

We consider a diffusion approximation $X_t^1 = X_0 + \eta t + \sigma W_t$ to the surplus process of an insurer. The insurer can buy proportional reinsurance continuously. The reinsurance premium is calculated by an expected value principle. This yields the surplus process

$$X_t^b = X_0 + \int_0^t [\eta - (1 - b_s)\theta] \,\mathrm{d}s + \int_0^t b_s \sigma \,\mathrm{d}W_s \;.$$

In order that the problem considered in this talk is not trivial, we assume $\theta > \eta$.

The drawdown process is defined as the difference to the historic maximum of the surplus

$$D_t^b = \max\{\sup_{s \le t}(X_s^b - X_t^b), x - X_t\} \;.$$

We here allow a starting drawdown x. Losing too much compared to the historic maximum sends a bad signal to the market and should therefore be prevented. Because a large drawdown cannot be prevented, we want to keep the time with large drawdown small. We therefore aim to minimise the weighted time in drawdown

$$V^b(x) = \mathbb{E}\Bigl[\int_0^\infty \mathrm{e}^{-\delta t} \mathbb{1}_{\{D_t^b > d\}} \,\mathrm{d}t\Bigr] \;,$$

where d is a critical level.

We show that it is possible to split the problem into the regions $\{D_t^b \le d\}$ and $\{D_t^b > d\}$. We then have to minimise, maximise respectively, the Laplace transform of the time to enter the other area. We solve the corresponding HJB equation explicitly. This gives an optimal strategy of feedback form, for which we find that the corresponding controlled process exists.

Keywords: drawdown; diffusion approximation; optimal proportional reinsurance; Hamilton–Jacobi–Bellman equation

References

[1] Leonie Violetta Brinker and Hanspeter Schmidli (2022), "Optimal discounted drawdowns in a diffusion approximation under proportional reinsurance." *Journal of Applied Probability*, **59**, to appear.

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