Coskewness under Dependence Uncertainty

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Abstract

In this paper, we study the impact of dependence uncertainty on coskewness. That is, we aim to maximize and to minimize $\mathbb{E}[(X-\mathbb{E}(X))(Y-\mathbb{E}(Y))(Z-\mathbb{E}(Z))]$, where X, Y and Z have given marginal distributions.

We derive these lower and upper bounds explicitly when the marginal distributions satisfy $F_X(\mathbb{E}(X)) = F_Y(\mathbb{E}(Y)) = F_Z(\mathbb{E}(Z)) = \frac{1}{2}$ and provide explicit expressions of the corresponding copulas that reach them. We then use these explicit bounds to introduce a notion of "standardized rank coskewness" so that it takes values in $[-1,\ 1]$, and it is invariant with respect to the marginal distributions.

Moreover, we provide a numerical method based on simulated annealing to estimate for all marginal distributions the risk bounds on coskewness and the corresponding copulas.

Keywords: Coskewness, Copula, Risk bounds, Marginal distribution, Multivariate model.

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