

Randomization and the Valuation of Guaranteed Minimum Death Benefits

Griselda Deelstra ^{*1} and Peter Hieber ^{†2}

¹ Department of Mathematics, Université Libre de Bruxelles, Boulevard du Triomphe, 1050 Brussels, Belgium.

² Department of Actuarial Science, Faculty of Business and Economics, University of Lausanne, 1015, Lausanne, Switzerland.

Abstract

Randomization describes the idea to replace known quantities, like, for example, model parameters, by random variables. In derivative pricing, randomizing the maturity time often leads to much more convenient valuation formulas that well approximate its non-random counterpart. The same valuation concept can be applied if payment dates are per se random - as is the case in insurance. We focus on death-linked contingent claims paying a random financial return at the time of death. In the very general case where financial returns follow a regime switching model with two-sided phase-type jumps, the advantage of our approach is that prices are closed-form, avoiding numerical Fourier inversion or Monte-Carlo simulation. Further, the distribution of remaining lifetime is approximated by a series of Erlang distributions, respectively a Laguerre series expansion, whose capability to fit the tail of the observed mortality data turns out to be much better than the commonly used series of exponential distributions.

Keywords: variable annuities, death benefits, Erlangization, life and pension insurance, randomization, regime switching, phase-type distributions, Erlang distribution.

References

- [1] Deelstra, G., Hieber, P. (2022). “Randomization and the Valuation of Guaranteed Minimum Death Benefits” *Working Paper*.
- [2] Deelstra, G., Latouche, G., & Simon, M. (2020). “On barrier option pricing by Erlangization in a regime-switching model with jumps.” *Journal of Computational and Applied Mathematics*, vol. **371**.

*E-mail address: griselda.deelstra@ulb.be

†E-mail address: peter.hieber@unil.ch

- [3] Gerber, H. U., Shiu, E. S., & Yang, H. (2012). "Valuing equity-linked death benefits and other contingent options: a discounted density approach." *Insurance: Mathematics and Economics*, vol. **51**(1), pp. 73-92.
- [4] Siu, C. C., Yam, S. C. P., & Yang, H. (2015). "Valuing equity-linked death benefits in a regime-switching framework." *ASTIN Bulletin: The Journal of the IAA*, vol. **45**(2), pp. 355-395.