Joint lifetime modeling with mIPH distributions

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Abstract

Acyclic phase-type (PH) distributions have been a popular tool in survival analysis, thanks to their natural interpretation in terms of ageing towards its inevitable absorption. It is therefore interesting to consider the potential of multivariate PH distributions for the modelling of joint human lifetimes. In the univariate case, it was recently demonstrated in [2] that introducing time-inhomogeneity into the stochastic construction of the PH distribution can greatly reduce the number of needed dimensions for an adequate fit of mortality rate curves.

In this talk, we will consider an extension to the bivariate setting for the modelling of joint lifetimes. In contrast to previous models in the literature that were based on separate estimation of the marginal behavior and the dependence structure through a copula, using a new time-inhomogeneous version of a multivariate PH class (mIPH) we show how to model joint lifetimes without separating the estimation of marginal and dependence properties. This also leads to a more natural causal interpretation of the resulting model. The main advantages of this mIPH class are its flexibility, denseness on the positive orthant and, unlike the MPH* class, statistical tractability. The idea is to introduce the dependence between the different components through a sharing of the initial state. This creates dependence, while still maintaining favourable properties of independence. We provide additional attributes of the mIPH class and an adapted estimation procedure that allows for right-censoring and covariate information. We show that initial distribution vectors can be tailored to reflect information that may affect the dependence of random variables, using multinomial regressions to predict the influence of covariates on starting probabilities. Moreover, we highlight the flexibility and parsimony in terms of needed phases that is introduced by time-inhomogeneity. We illustrate our results on the famous dataset of joint lifetimes of [8], where 10 phases turn out to be sufficient for a reasonable fitting performance. We finally interpret the results in terms of a correlated ageing mechanism of joint lifetimes that goes beyond a statistical fit.

Keywords: Mortality Modelling, multivariate PH distributions, censoring, EM algorithm

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