Risk-sharing in equity-linked insurance products: Stackelberg equilibrium between an insurer and a reinsurer

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Abstract

Equity-linked insurance products often have capital guarantees (see, e.g., [3, 4]). Common investment strategies ensuring these guarantees are challenged nowadays by low interest rates. Thus, we study an alternative strategy when an insurance company shares financial risk with a reinsurance company. We model this situation as a Stackelberg game, which is a bi-level optimization problem. As the number of reinsurance companies is significantly lower than the number of primary insurance companies ([1]) and reinsurance companies have a rather dominant position ([2]), the reinsurer is the leader in the Stackelberg game that we consider. The reinsurance company maximizes its expected utility by selecting its optimal investment strategy and a safety loading in the reinsurance contract it offers to the insurer. The reinsurer can assess how the insurer will rationally react on each action of the reinsurer. The insurance company is the follower and maximizes its expected utility by choosing its investment strategy and the amount of reinsurance the company purchases at the price offered by the reinsurer. In this game, we derive the Stackelberg equilibrium for general utility functions. For power utility functions, we calculate the equilibrium explicitly and find that the reinsurer selects the largest reinsurance premium such that the insurer may still buy the maximal amount of reinsurance. Since in the equilibrium the insurer is indifferent in the amount of reinsurance, in practice, the reinsurer should consider charging a smaller reinsurance premium than the equilibrium one. Therefore, we propose several criteria for choosing such a discount rate and investigate its wealth-equivalent impact on the utilities of both parties.

This talk is based on the joint work with M. Hinken and R. Zagst [5].

Keywords: risk sharing, portfolio optimization, Stackelberg game, insurance, reinsurance

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