Distributionally Robust Estimation of Moments under Partial Ambiguity

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Abstract

The model uncertainty issue is pervasive in virtually all applied fields but especially critical in insurance and finance. To hedge against the uncertainty of the underlying probability distribution, which we refer to as ambiguity, distributionally robust optimization (DRO) has been well developed during recent years. However, this approach often yields results that are overly conservative. We argue that in most practical situations a generic risk is realized from multiple scenarios. In some ordinary scenarios, the risk may be subject to negligible ambiguity so that it is safe to trust the reference distributions, and hence we need to apply DRO only to the other scenarios where the ambiguity presents and matters. We implement this idea into robust estimation of the moments of a risk in the hope to alleviate the over-conservativeness issue. Note that under this consideration the ambiguity exists in both the scenario indicator and the risk realization in a corresponding scenario, leading to a twofold ambiguity issue. We employ the Wasserstein distance to construct an ambiguity ball and then carefully disentangle the ambiguity along the two folds so as to link our robust estimation problem to established DRO results. Our main result is a closed-form robust estimate for the moments. We also conduct numerical studies to illustrate the benefit of our consideration of partial ambiguity.

Keywords: Robust estimation; Scenario analysis; Partial ambiguity; Distributionally robust optimization; The Wasserstein distance

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