Valuation of Guaranteed Minimum Benefits under Self-exciting Stochastic Mortality

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Guaranteed Minimum Benefits (GMB) are contracts that provide the policyholder with a minimum guaranteed benefit. It guarantees a minimum value to the insured in some prescribed conditions which usually depend on the health of the insured. Such insurance contracts allow for flexibility in participating in the financial market by investing the premiums into it. In this way, the policyholder can make decisions on how and where to invest their premiums. There are many types of GMBs. Perhaps, the most well-known ones are Guaranteed Minimum Maturity Benefit (GMMB) and Guaranteed Minimum Death Benefit (GMDB), where the former protects the value of the annuity from market fluctuations and secures a minimum retirement benefit for the insured upon survival and the latter is a guarantee that pays out a minimum benefit upon death during the term of the contract.

The pricing of such equity-linked contracts requires the modelling of risk factors such as financial risk, biometric risk and interest rate risk. In this work we develop a framework for pricing such contracts under a biometric scenario where periods of heightened mortality is present due to e.g. extreme temperatures or contagious deadly illness modelled by so-called self-exciting processes. While we still model market and interest rate risk, we focus on a model for mortality that has some sort of contagion described by a Hawkes process, see [1]. Hawkes processes are widely used to model clustering effects in both finance and nature, such as earthquakes or volatility clusters. Here, we use the self-exciting feature of Hawkes processes to capture the contagion effects of deaths in, for instance, a pandemic scenario. At the same time, we still wish to derive tractable pricing formulas despite Hawkes processes not being Markovian per se, which we achieve by exploiting their affine structure. For an example in a classical Markovian setting, with regimes see e.g. [2].

In a summary, we model market and interest rates with classical models and mortality risk with a stochastic version of Gompertz-Makeham law of mortality where we incorporate a self-exciting component, meaning that increases in number of deaths incites further deaths. This could typically be the behaviour of mortality under an epidemic or pandemic. We also include cross-correlations in our models between the risk factors, which is a sensible assumption.

More specifically, we consider the following modelling framework for μ_t the force of mortality at time t for a specified generation and assume that it evolves as

$$d\mu_t = m(\overline{\mu}_t - \mu_t)dt + \zeta dW_t + \eta dL_t, \tag{1}$$

$$d\lambda_t = \beta(\overline{\lambda} - \lambda_t)dt + \alpha dL_t, \tag{2}$$

$$L_t = \sum_{i=1}^{N_t} J_i \tag{3}$$

where the jumps J_i are assumed to be i.i.d. and N is a counting process with stochastic, self-exciting intensity λ_t, $m, \zeta, \eta \in \mathbb{R}$, $m, \zeta, \eta > 0$ and W is a Brownian motion correlated to the bond and financial market. Here, $\overline{\mu}$ is a deterministic trend (e.g. Gompertz-Makeham model). The parameter m is assumed to be positive so that μ is reverting to a mortality trend modelled by $\overline{\mu}$. The process μ should not be confused with the central mortality for a given age x and calendar year t, i.e. often denoted by $m(x,t)$, but it is related. When computing probabilities we look at $\mu(x+t,t)$ where x is the age of the individual at initial time and u is the calendar year. Here, we model $\mu_t = \mu(x+t,t)$ for a given fixed age x as in (1).

In summary, the overall aim of the this research is to capture the self-exciting feature of e.g. pandemics using Hawkes processes and derive reserving formulas for equity-linked insurance contracts. More concretely, we derive closed formulas for longevity bonds linked to this type of stochastic mortality and some other GMB products. Thereafter, we conduct some simulation and calibration study of the proposed model. In particular, we conduct an empirical study casting a glance at Spanish mortality data from 2012-2021 and give empirical evidence of the existence of a self-exciting feature during the coronavirus pandemic in 2020-2021. We use an iterative peaks over threshold method (POT) to adjust the model. We study all age groups and find out that the coronavirus pandemic in Spain was mostly self-exciting for young age groups than older. We look at the impact a pandemic has on the prices of insurance contracts and what parameters of the model in (1) and (2) are most sensitive.

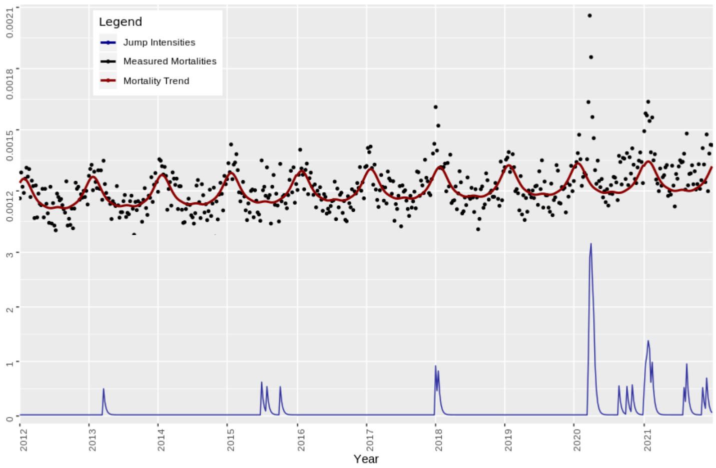

Figure 1: Spanish mortality data for age group 15-64 in the period 2012-2021. We observe a clear self-exciting effect during the pandemic period 2020-2021.

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