Valuation of guaranteed minimum benefits under generalised regime-switching models using Fourier Cosine method

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Abstract

This paper presents a flexible valuation approach for variable annuity (VA) contracts embedded with guaranteed minimum maturity benefit (GMMB) riders whose underlying fund dynamics are governed by a general regime-switching framework. Pricing of such contracts has been extensively studied in existing literature, with the bulk of which premised around the geometric Brownian motion (GBM) framework. This assumption significantly simplifies the valuation process making it possible to derive analytical pricing formulas [2]. Recently, there has been a gradual shift from the classic GBM paradigm towards more realistic frameworks. Depending on the types of underlying fund dynamics, recent literature on VA pricing can classified into three categories. The first focuses on incorporating stochastic interest rate and stochastic mortality risk in valuing VA riders [3] whilst the second direction involves modelling underlying fund dynamics using Lévy processes [1]. However, neither diffusion-driven models nor Lévy-based models are able to capture the structural changes observed in the financial markets. [7] consider vanilla options under regime-switching models which prove to be ideal candidates for characterising long-dated contracts like VAs. As such, the third research direction utilises regime-switching models to capture the macroeconomic risk inherent in markets with [6] utilising the Fourier space time-stepping algorithm in pricing and hedging of a variety of guaranteed minimum benefit riders under the regime-switching and stochastic mortality models.

As foundational to VA riders encountered in practise, this work considers pricing of GMMB riders with three distinct characterisations namely; fixed, rolled up, and geometric average guarantees. We develop a novel class of generalised regime-switching (GRS) models which extend the classical regime-switching (CRS) models [4]. Under a CRS model, after regime shifts only the values of model parameters will change and the model structure remains the

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same. However, in a GRS model, upon regime shifts not only the values of model parameters vary, but also the model structure may as well change.

Our framework is general enough as it only requires the characteristic function of the log-stock price to satisfy a structural assumption and do not need to specify the price dynamics from the very beginning. We utilise the characteristic functions of underlying processes to devise a Fourier Cosine (COS) algorithm [5] for VA pricing. The implementation of the COS method is non-trivial. We establish a set of ingenious regime-dependent bounds that are used in the approximation procedure of the COS method. To find such regime-dependent bounds, we derive a generalised characteristic function allowing for a Markov-modulated domain. This innovation yields a highly efficient COS algorithm which outperforms Monte Carlo simulation. Sensitivity analyses of the VA contract values with respect to various model parameters are performed where we note that as the likelihood of regime shifts increases, the difference of VA contract prices with different initial regimes will diminish which is consistent with financial intuition. GRS models are more suitable for VA pricing as maturities usually spans over several decades.

Keywords: Variable annuities; GMMB; COS method; Generalised regime-switching model

References

[1] J. Alonso-García, O. Wood, O. and J. Ziveyi (2018), "Pricing and hedging guaranteed minimum withdrawal benefits under a general Lévy framework using the COS method." *Quantitative Finance*, vol. 18 no.(6), pp. 1049-1075.

[2] D. Bauer, A. Kling, J. Russ (2008), "A universal pricing framework for guaranteed minimum benefits in variable annuities." *ASTIN Bulletin*, vol. 38 no.(2), pp. 621-651.

[3] T.S. Dai, S.S. Yang, L.C. Liu (2015), "Pricing guaranteed minimum lifetime withdrawal benefits with various provisions under investment, interest rate and mortality risks." *Insurance: Mathematics and Economics*, vol. 64, pp. 364-379.

[4] R.J. Elliott, L. Chan, T.K. Siu (2005), "Option pricing and Esscher transform under regime switching." *Annals of Finance*, vol. 1, pp. 423-432.

[5] F. Fang, C.W. Oosterlee (2008), "A novel pricing method for European options based on Fourier-cosine series expansions." *SIAM Journal on Scientific Computing*, vol. 31 no.(2), pp. 826–848.

[6] K. Ignatieva, A. Song, J. Ziveyi (2016), "Pricing and hedging of guaranteed minimum benefits under regime-switching and stochastic mortality. " *Insurance: Mathematics and Economics*, vol. 70, pp. 286-300.

[7] Y. Shen, K. Fan, T.K. Siu (2014), "Option valuation under a double regime-switching model." *Journal of Futures Markets*, vol. 34 no.(5), pp. 451-478.