Multivariate matrix-exponential affine mixtures and their applications in risk theory

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Abstract

Consider a class of positive M-variate random vectors whose multivariate density function can be written as

$$f(x_1, \ldots, x_M) = \sum_{i_1,\ldots,i_M \in \{1,\ldots,L\}} p(i_1, \ldots, i_M) f_{i_1}(x_1) \cdots f_{i_M}(x_M),$$

where $p(\cdot)$ is such that $\sum_{i_1,\ldots,i_M \in \{1,\ldots,L\}} p(i_1, \ldots, i_M) = 1$, and each f_j is an univariate matrix-exponential density function of the form $f_j(x) = \alpha_j e^{T_j x} t$. We call such a class of densities multivariate matrix-exponential affine mixtures. Univariate matrix-exponential distributions generalize the class of phase-type distributions, which have been extensively been used in the risk theory literature due to their flexibility and tractability. In this talk, we show that multivariate matrix-exponential affine mixtures inherit these qualities of their univariate counterpart. In particular, we show various attractive properties such as such as closure under size-biased Esscher transform, order statistics, residual lifetime and higher order equilibrium distributions. This allows for explicit calculations of various actuarial quantities of interest. The results are applied in a wide range of actuarial problems including multivariate risk measures, aggregate loss, large claims reinsurance, weighted premium calculations and risk capital allocation. Furthermore, a multiplicative background risk model with dependent risks is considered and its capital allocation rules are provided as well.

Keywords: Matrix-exponential distribution; Multivariate affine mixtures; Risk measures; Capital allocation; Multiplicative background risk models.

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