Matrix methods in insurance risk

Mogens Bladt [*1]

[1]Department of Mathematical Sciences, University of Copenhagen

Abstract

The talk aims to provide an overview of the matrix techniques in both life and non–life insurance, wherein notably matrix–exponential methods will show their usefulness, tractability, and generality.

The starting point is time–inhomogeneous phase–type distributions (IPH), defined as the distributions of the time until absorption of (time–inhomogeneous) Markov jump processes. They provide a dense class of distributions on the positive reals, which means that any non–negative distribution may be approximated arbitrarily well by an IPH. Another essential feature is the independent specification of their tail behavior, ranging from the light tails in life insurance (e.g., Gompertz or Weibull) to the heavy tails of claims appearing in non–life insurance (e.g., Pareto).

While IPH can be used in a parsimonious way in non–life insurance to estimate heavy-tailed claims, in life insurance, it is rather the distribution of a reward structure imposed on the underlying Markov process that is of interest. For example, the mean of this reward distribution is the technical (prospective) reserve. Higher-order moments can be obtained similarly.

Rather than providing an in-depth mathematical treatment of one particular application, we consider several examples from both life and non-life insurance where the use of matrices, instead of individual variables, enables us to provide compact and explicit formulas and thereby greater transparency.

Keywords: Phase–type distribution; Product integral; EM–algorithm; heavy–tails; reserve; Thiele; Gram-charlier expansion; functions of matrices.

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[*]E-mail address: bladt@math.ku.dk