l_1-Regularization in Portfolio Selection with Machine Learning

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Abstract

Regularization parameter selection is one of the most essential tasks in solving large-scale ill-posed problems. Problems of this kind arise in various applications, and generally, their solution requires some regularization; that is, the problem is substituted by a related one with better numerical properties. A common approach is to add a penalty term that enforces uniqueness and stability. The penalty term is controlled using a regularization parameter. It must realize a trade-off between fidelity to data and regularization. If the regularization parameter is too small, the model has numerical features similar to the unregularized one; on the other hand, the solution doesn't fit the original model if it is too big. In the context of Portfolio optimization, different regularization techniques have been suggested for the mean-variance Markowitz model [2]. Among these, l_1 penalization has been considered. This is an effective technique to obtain sparse portfolios that allow the investor to reduce both the number of positions to be monitored and the holding costs [1, 3, 4]. l_1 regularization parameter selection is often based on problem-dependent criteria and related to iterative empirical estimates, requiring a high computational cost. In [1] a least-angle regression algorithm is presented, which starts from large values and proceeds by decreasing them. In [3, 4] authors propose an iteration procedure based on Bregman iteration method, which includes an adaptive rule for the selection of the regularization parameter, respectively, in the single-period and multi-period framework.

In this talk, we explore the use of supervised Machine Learning (ML)techniques for the automatic selection of the regularization parameter in the multi-period portfolio selection model presented in [4]. ML provide methods which are data-driven; it is actually extensively applied to Finance, in particular to the portfolio selection problem [5]. The aim of ML is to develop algorithms which can learn and progress over time and can be used for predictions. In particular, the goal of supervised learning is to predict the value of one or more outputs for a set of inputs. Thus, we aim at approximating $f : \mathcal{X} \subseteq \mathbb{R}^q \longrightarrow \mathcal{Y} \subseteq \mathbb{R}^t$ with a function

$$f_{\boldsymbol{\theta}}(\mathbf{x}) = f_{\boldsymbol{\theta}}(\mathbf{x}, \boldsymbol{\theta}),$$

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where $f_{\boldsymbol{\theta}} : \mathcal{X} \longrightarrow \mathcal{Y}$ is usually nonlinear and $\boldsymbol{\theta} \in \mathbb{R}^q$ is a large set of unknown parameters. The learning phase uses a training set to produce a set of parameters $\boldsymbol{\theta}$ that minimize a loss (or cost) function $\mathcal{L}$ that measures the accuracy of the predicted $f_{\boldsymbol{\theta}}(\mathbf{x})$ with respect to $f(\mathbf{x})$.
In this talk, we consider the Neural Networks (NN) that have become particularly popular among Machine Learning methods and we discuss the application of NN to select the regularization parameter in l_1 regularized portfolio optimization. We compare the solutions of the portfolio selection problem obtained with the adaptative rule presented in [4] with the one obtained using the NN approach. Results show the effectiveness of our approach. The NN based approximation seems to accurately capture the relation between the selected features and the optimal regularization parameter. Optimal portfolios exhibit satisfying financial properties. Moreover, results show that the proposed algorithm often outperforms an existing method for the same problem.

Keywords: Deep Learning; Multi-period portfolio optimization; l_1-norm; Split Bregman.

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