Assessing model uncertainty for log-symmetric distributions

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Abstract

Model-based decisions are susceptible to model uncertainty that arises from the inadequacy of the adopted model. One natural way to address model uncertainty is to compare the adopted model with the worst-case and best-case models (i.e., models that yield a maximum resp. a minimum value for some chosen risk measure) that are consistent with a set of trusted assumptions. Even though it is common in actuarial and financial modeling to deal with risks whose distributions become symmetric after a log transformation, the literature (to the best of our knowledge) did not consider this assumption in assessing model uncertainty.

In this talk, we present upper and lower bounds for the Value-at-Risk of log-symmetric risks under two sets of (trusted) assumptions. First, we consider the case in which the log-transformed random variable is known to have a unimodal and symmetric distribution with a known mean and a known maximum variance. The bounds, in this case, are sharp and significantly improve over those available in the literature for all probability levels lower than some extreme value. However, the practical issue of this case is that the upper bound increases dramatically at extreme probability levels.

Second, we consider the case in which the random variable at hand is known to be log-symmetric and has a unimodal distribution with a known median, a known maximum mean, and a known second moment. For high probability levels, we could derive an upper bound that is sharp and stable for extreme probability levels and consequently offer a significant improvement over the bound of the first case (and the findings in the literature) for high probability levels. Indeed, this upper bound can be extended to the cases where the first moment is unknown but finite and the second moment is unknown and possibly infinite.

Our approach for deriving the extreme models consists of two steps. In a first step, we use some classical results on stochastic ordering to reduce the optimization problem to a parametric one, which in a second step can be solved using standard methods.

Our findings are illustrated using real-world datasets: a automobile insurance claims dataset and a general liability claims dataset (heavy-tailed).

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Keywords: Value-at-Risk; risk bounds; model uncertainty; log-symmetry; unimodality.

Acknowledgements: The authors gratefully acknowledge funding from Fonds Wetenschappelijk Onderzoek (grants FWOODYSS11 and FWOSB73).