Risk of ruin in SIS type epidemics

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Abstract

SIS (Susceptible-Infected-Susceptible) epidemic models describe the spread of an infectious disease in a closed population of N individuals, partitioned into two groups: the class of susceptibles contains the individuals free of the disease and the class of infectives those who carry the infectious agent. The dynamics is as follows: when a susceptible gets infected, he becomes contagious for a random duration and can transmit the disease to the susceptibles. Then he recovers and becomes exposed to the disease (i.e. susceptible) again. The epidemic terminates as soon as there are no more infected individuals in the population.

A classical SIS model is the so-called logistic epidemic, in which each infective remains contagious for an exponential duration before recovering, and where the contamination rate per infected individual is proportional to the number of susceptibles. Let S_t and I_t be the number of susceptibles and infectives present at time t. Since $S_t + I_t = N$ for all t, the logistic epidemic can be described through the process $\{S_t\}$ only, which is a Markovian birth-and-death process with possible instantaneous transitions:

$$\begin{aligned} s &\to s-1, \quad \text{at rate } \beta s(N-s), \\ s &\to s+1, \quad \text{at rate } \mu(N-s), \end{aligned} \tag{1}$$

for some parameters $\beta, \mu > 0$.

In this talk, we combine risk theory and epidemiology by studying the problem of possible ruin when an insurance company provides a coverage against the risk of disease to a population subject to an SIS epidemic. The epidemic dynamics is an extension of the classical logistic model, where (1) is replaced by the more general rates:

$$\begin{aligned} s &\to s-1, \quad \text{at rate } \beta(s), \\ s &\to s+1, \quad \text{at rate } \mu(s), \end{aligned}$$

with $\beta(0) = 0$, $\mu(N) = 0$, $\beta(N) = 0$ and where the other rates are strictly positive. Here, $\beta(s)$ represents the global infection rate on s susceptibles and $\mu(s)$ the global recovery rate

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for $i = N - s$ infectives. We assume that the premiums are continuously collected from the susceptibles while the care costs are reimbursed to the infected individuals via an annuity (i.e. the company reimburses the care of infectious people at some rate for the duration of the treatment) or via a lump-sum benefit (i.e. a random compensation amount is paid to each infective as soon as they recover from the disease).

We first determine the joint distribution of several variables that influence the cost of insurance coverage for an SIS type epidemic: the duration of the epidemic, the total number of contaminations and the total cost of the epidemic from the insurer's point of view. Next, we analyse the process $\{(S_t, I_t, R_t)\}$ where R_t is the reserve of the insurance company at time t. We mainly focus on the ruin of the insurance company when it occurs before the infection is over: we use matrix-analytic methods to determine the joint distribution of the time of ruin, the total number of contaminations and the population state at the time of ruin.

Finally, we consider a variant of the model, where we follow the risk process over a large time scale so that it can now alternate between normal periods (without disease) and epidemic episodes. This leads us to introduce a regenerative extension of the model using a Brownian reserve process. We obtain the joint distribution of the ruin statistics above, together with the number of epidemics triggered before ruin.

Keywords: Birth-and-death epidemic model; Insurance coverage; Ruin statistics; Brownian regenerative reserve process.

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